How the Nature of Product Differentiation Affects Procurement Competition*

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Abstract
This paper compares the equilibrium outcomes from a recently developed model of procurement competition with differentiated products to those from two analytically tractable models that might naturally be considered suitable proxies. The models differ in what sellers know about the buyer’s preferences for their products, and the newer model must be solved numerically. The three models yield substantially different prices and payoffs, and they exhibit qualitatively different comparative statics. These findings caution against using the analytically convenient models when the newer model is empirically appropriate. Reinterpreting the models and results also provides insights related to earlier analyses of price discrimination in oligopoly.

1 Introduction
Sellers in procurement markets frequently offer differentiated products. For example, the well-documented competition between Airbus and Boeing to supply planes to Iberia Airlines was influenced by Iberia’s preferences for both sellers’ products.1 Related stories illustrate that product differentiation affects the procurement processes of companies like Hewlett-Packard, IBM, Kaiser Permanente, Nissan, Pfizer, and Sun Microsystems.2

A natural way to model such procurement environments combines two commonly used approaches in an empirically relevant way. The first is that the sellers’ products are horizontally differentiated, as is standard in discrete choice models such as those surveyed by Anderson, et al. [1992]. The second is that the sellers’ production costs are privately known, as is standard in procurement models with homogeneous products such as Holt [1980]. The model exhibits what I refer to as private preferences: The buyer knows how much it values seller i’s product, but all sellers are uncertain about that value. The procurement format is what

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Engelbrecht-Wiggans, et al. [2007] refer to as a buyer-determined mechanism: Sellers simultaneously offer prices to the buyer, and the buyer purchases at the offered price from the seller whose price offer and value combine to give the buyer its greatest positive payoff. Thomas and Wilson [2011] introduce this model to form the basis for their experimental comparison of different procurement mechanisms.

Unfortunately the private preferences model does not appear to be analytically tractable, so in this paper I use numerical methods to compare its outcomes to analytically derived ones from two models that one might consider to be natural proxies. The first exhibits semi-private preferences: The buyer and each seller $i$ know how much the buyer values seller $i$’s product, but all other sellers are uncertain about that value. The second exhibits homogeneous preferences: The buyer has the same commonly known value for all sellers’ products.

One might anticipate some similarity among the three settings’ equilibrium outcomes. The private and semi-private preference settings differ from each other only in the amount of symmetrically distributed private information the sellers have. The homogeneous preference setting is constructed so expected total surplus from any particular transaction is the same as in both settings with differentiated products.

I find that pricing behavior differs consistently across all three settings. On average sellers set higher prices with semi-private than with homogeneous preferences, which reflects the sellers’ response to the increased variance in the surplus available from the individual trades. Prices are even higher with private preferences, which reflects the sellers’ response to their increased uncertainty as to whether they are the buyer’s surplus-maximizing trading partner.

Despite the consistency of the price changes across the three settings, I find that changes in the buyer’s and sellers’ expected payoffs exhibit no clear pattern. For example, whether the buyer’s expected payoff increases or decreases when moving from homogeneous to semi-private preferences depends on the number of sellers, but the buyer’s expected payoff always falls when moving from semi-private to private preferences. Likewise, across settings there is no obvious pattern in the sellers’ expected payoffs, or in the ratio of the buyer’s to the sellers’ expected payoffs.

The qualitative differences in outcomes across the three settings highlight the importance of appropriately specifying the information available to market participants. This is especially important because the private preference setting is empirically relevant but difficult to analyze, and for the reasons described earlier one might be inclined to approximate its equilibrium outcomes using the analytic results with homogeneous or semi-private preferences. My results demonstrate that those equilibria are not suitable proxies, because payoffs in the different settings exhibit qualitatively different comparative statics with respect to the models’ structural parameters. Therefore, if the private preferences model is empirically appropriate, then using the simpler modeling approaches can give incorrect predictions about issues of interest, such as sellers’ entry or investment decisions.

Section 2 presents the three procurement models, and Section 3 compares their equilibrium outcomes. Section 4 relates the results to those from existing models of price discrimination in oligopoly, which is relevant because the semi-private and private preference settings can be interpreted to be settings with and without third-degree price discrimination. Section 5 briefly concludes.

2 Procurement with Horizontal Product Differentiation

Consider $N$ risk-neutral sellers competing to supply one unit of an indivisible good to a risk-neutral buyer. Two types of uncertainty are partially resolved before the sellers make simultaneous price offers to the buyer.
First, each seller $i$ draws its production cost $c_i$ independently from the commonly known and differentiable cumulative distribution $F(c)$. The cost draw $c_i$ is known only by seller $i$. Assume that $F$ has a density $f$ that is strictly positive on the interior of its support $[\underline{c}, \overline{c}]$, with $0 \leq \underline{c} < \overline{c}$.

Second, the buyer draws its value $v_i$ for each seller’s product independently from the commonly known and differentiable cumulative distribution $G(v)$. The value draw $v_i$ is known by the buyer, and may be known by seller $i$. Assume that $G$ has a density $g$ that is strictly positive on the interior of its support $[\underline{v}, \overline{v}]$, with $0 \leq \underline{v} < \overline{v}$. Also assume that $\underline{c} < \overline{v}$, so there are gains from trade for at least some realizations of costs and values.

The buyer purchases, at the offered price, from the seller whose price offer gives the buyer its greatest positive payoff. If the buyer purchases from seller $i$ at price $p_i$, then the buyer’s payoff is $v_i - p_i$, seller $i$’s payoff is $p_i - c_i$, the other sellers’ payoffs are 0, and total surplus is $v_i - c_i$. Engelbrecht-Wiggans, et al. [2007] and Shachat and Swarthout [2010] note that this buyer-determined mechanism is widely used in industrial procurement. It is analogous to a first-price auction with homogeneous preferences: the seller offering the most surplus to the buyer wins and is paid the price that it offered.

A seller’s strategy is a mapping to its price offer from its cost realization (and possibly its value realization). Seller $i$ with cost $c_i$ selects its price $p_i$ to maximize $(p_i - c_i) \Pr[p_i \text{ wins}]$, where

$$\Pr[p_i \text{ wins}] = \Pr[v_i - p_i \geq \max_{j \neq i} 0, \max [v_j - p_j]] .$$

The probability that $p_i$ wins depends on the distributions $F(c)$ and $G(v)$, what sellers know about the buyer’s values for their products, and the price-setting strategies used by seller $i$’s rivals. Given that the same distributions $F$ and $G$ apply to all sellers, I consider only symmetric equilibria in which all sellers use the same strategy.

Although the model considers competition for a single buyer’s business, it essentially is a linear random utility model like those commonly used in empirical analyses of markets with horizontally differentiated products.3 These analyses typically assume atomistic buyers and complete information about costs, but the extension to large buyers and incomplete information is a natural one. The horizontal differentiation interpretation is appropriate here even though a particular buyer rank-orders the sellers’ products in terms of the values, because different buyers have different preferences based on their value realizations.

The model also is a scoring auction of the sort introduced in Che [1993]. The specific scoring rule considered uses the buyer’s true preferences, which Che [1993] notes may be more appropriate than scoring rules that distort the weight attached to the quality dimension.4 The environment I consider differs in two main ways from the environment commonly used in the literature on scoring auctions: a seller does not tailor its product to the buyer’s preferences, and a seller’s cost is independent of the buyer’s value for the seller’s product. Both assumptions are commonly associated with settings featuring horizontal product differentiation, but the appropriateness of either is an empirical matter specific to the market in question.

### 2.1 Procurement with Semi-Private Preferences

Suppose it is common knowledge that seller $i$ knows $v_i$, and that all other sellers have only beliefs about $v_i$ given by $G(v)$. Assuming that rival sellers use the price-setting function $\hat{p}(c, v)$, seller $i$ maximizes its

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4 Scoring rules that distort on quality assume the buyer can commit to consummating a transaction that is inferior to another available trade.
interim expected payoff

\[ \pi_i^{SP}(p_i|c_i, v_i, \hat{p}) = (p_i - c_i) \int_{W^{SP}(p_i, v_i, \hat{p})} \left( \prod_{j \neq i} dF(c_j) dG(v_j) \right), \]

where \( W^{SP}(p_i, v_i, \hat{p}) = \{ (c_j, v_j)_{j \neq i} : v_i - p_i \geq \max [0, \max_{j \neq i} [v_j - \hat{p}(c_j, v_j)] ] \} \). \( W^{SP}(p_i, v_i, \hat{p}) \) is the set of rivals’ costs and values for which seller \( i \) wins when its value is \( v_i \) and it sets price \( p_i \).

Deriving analytic solutions is straightforward with semi-private preferences once one recognizes that each seller, relative to rival sellers, has private information about the total surplus its trade with the buyer creates. One simply translates a standard first-price procurement auction, in which sellers have private cost information and compete by offering prices, to an auction in which sellers have private total surplus information and compete by offering surplus.

Specifically, consider the cumulative distribution of total surplus \( H(s) \), where \( s \equiv v - c \). With the assumptions on \( F \) and \( G \), \( H(s) \) is commonly known and has a density \( h \) that is strictly positive on the interior of its support \([v - c, v - c]\). For a given total surplus \( s_i \), seller \( i \) selects an amount of surplus \( \varphi_i \equiv v_i - p_i \) to offer to the buyer. Assuming seller \( i \)’s rivals all use the strictly increasing offer function \( \hat{\varphi}(s) \), seller \( i \)’s interim expected payoff can be written as \( (s_i - \varphi_i) H \left( \hat{\varphi}^{-1}(\varphi_i) \right)^{N-1} \).

Shachat and Swarthout [2010] derive analytic solutions for the sellers’ equilibrium surplus-offer and price-setting functions. Although those authors assume \( c < v \), so that trade creates positive total surplus for all realizations of costs and values, the associated solutions are easily extended to include the case in which trade may not create positive total surplus. With suitable modification to account for the latter possibility, the sellers’ price-setting function in this game’s Bayesian Nash equilibrium is

\[ p^{SP}(c, v) = c + \int_0^{\max[0,s]} \left( \frac{H(x)}{H(s)} \right)^{N-1} dx. \]

If \( s < 0 \), then the seller offers the buyer negative surplus by setting its price equal to its cost, \( c \). This offer will surely be rejected. If \( s > 0 \), then the seller offers the buyer positive surplus by marking up its cost by an amount that depends on the total surplus \( s \) from the transaction, the distribution \( H(s) \), and the number of rival sellers.\(^5\) This equilibrium is efficient: The winning seller is the seller with whom trade yields the greatest total surplus, and trade occurs if and only if trade with at least one seller yields positive total surplus.

### 2.2 Procurement with Private Preferences

Suppose it is common knowledge that seller \( i \) does not know \( v_i \), and that all sellers have only beliefs about \( v_i \) given by \( G(v) \). Assuming that rival sellers use the price-setting function \( \hat{p}(c) \), seller \( i \) maximizes its interim expected payoff

\[ \pi_i^P(p_i|c_i, \hat{p}) = (p_i - c_i) \int_{W^{P}(p_i, \hat{p})} dG(v_i) \left( \prod_{j \neq i} dF(c_j) dG(v_j) \right), \]

\(^5\)The equilibrium price-setting function is unique for \( s > 0 \). For \( s \leq 0 \) sellers can make any offers that weakly exceed their cost, and those offers will surely be rejected. For my purposes I assume the sellers price in a specific way when \( s \leq 0 \).
where \( W^P (p_i, \tilde{p}) = \left\{ v_i, (c_j, v_j)_{j \neq i} \right\} : v_i - p_i \geq \max [0, \max_j [v_j - \tilde{p}(c_j)]] \). The difference from the interim expected payoff with semi-private preferences is the additional uncertainty about \( v_i \), which corresponds to an additional dimension of integration in \( \pi^P_i (p_i | c_i, \tilde{p}) \), relative to \( \pi^S_i (p_i | c_i, v_i, \tilde{p}) \).

The translation used with semi-private preferences does not work with private preferences, because seller \( i \) is unsure of the total surplus if it trades with the buyer. In addition, seller \( i \) does not know if its offer \( p_i \) gives the buyer positive surplus, and has additional uncertainty about its likelihood of offering more surplus than its rivals.

Deriving analytic solutions is hindered because the integral in \( \pi^P_i (p_i | c_i, \tilde{p}) \) that reflects the probability that \( p_i \) wins does not appear to have a convenient form, even for simple distributions of costs and values like the Uniform. While it is conceptually straightforward to understand the set \( W^P (p_i, \tilde{p}) \) over which the integration takes place in \( \pi^P_i (p_i | c_i, \tilde{p}) \), it is not obvious how to write it in a conveniently analyzed fashion. Roughly speaking, in the buyer-determined mechanism with homogeneous or semi-private preferences, for given strategies by rival sellers the expression \( \Pr [p_i \text{ wins}] \) measures a length such as \( F \left( \tilde{p}^{-1} (p_i) \right)^{N-1} \) or \( H \left( \tilde{\varphi}^{-1} (\varphi_i) \right)^{N-1} \). In a procurement auction with private preferences, for given strategies by rival sellers the expression \( \Pr [p_i \text{ wins}] \) measures a volume that is irregularly shaped because it is cut along different angles by different constraints. This problem is similar to the one encountered when the buyer has a random and secret reserve price, as in Elyakime, et. al [1994].

Because of the difficulty in deriving analytic solutions with private preferences, Thomas and Wilson [2011] numerically approximate the sellers’ equilibrium price-setting functions for specific distributions \( F(c) \) and \( G(v) \). They use an iterative procedure in which seller 1’s rivals are assumed to use a particular price-setting function, \( p_0(c) \). Seller 1’s best-response to \( p_0(c) \) is determined, denote it \( p_1(c) \). Assuming seller 1’s rivals use \( p_1(c) \), seller 1’s best-response to \( p_1(c) \) is determined. The process continues until the difference is sufficiently small between the rivals’ price-setting function \( p_{i-1}(c) \) and seller 1’s best-response \( p_i(c) \). The Appendix provides additional details about this procedure.

Unlike with semi-private preferences, equilibrium can be inefficient with private preferences. The buyer might not trade with the seller with whom trade creates the greatest positive total surplus, or trade might not occur even if there is a trade that creates positive total surplus.

### 2.3 Procurement with Homogeneous Preferences

Finally, consider the standard procurement scenario in which the buyer has the same commonly known value for all sellers’ products, and that value weakly exceeds \( \overline{c} \). This homogeneous preference setting can be considered a special case of both the semi-private and private preference settings in which the variance of the \( v_i \) is zero. As shown originally in Holt [1980], the sellers’ price-setting function in this game’s Bayesian Nash equilibrium is

\[
p^H(c) = c + \int c \left( \frac{1 - F(x)}{1 - F(c)} \right)^{N-1} dx.
\]

### 3 Comparing Procurement Outcomes

Given that the setting with private preferences at present can be solved only numerically, to compare procurement outcomes from all three settings I must specify the distributions \( F(c) \) and \( G(v) \) of costs and values. For simplicity I use the values of \( N \) and the distributions used in the experiments from Thomas and Wilson [2002, 2011]. \( N \) is either 2 or 4, the sellers’ costs are distributed Uniformly from 0 to 600, and the buyer’s
values are distributed Uniformly from 300 to 900. Their earlier experiment corresponds to the homogeneous preference setting, with the buyer’s value for each seller’s product commonly known to be 600. Their later experiment corresponds to the private preference setting, with the buyer’s expected value for each seller’s product equal to 600.

With semi-private and private preferences, total surplus \( s = v - c \) ranges from -300 to 900, the density \( h(s) \) is triangular, and the distribution \( H(s) \) is

\[
H(s) = \begin{cases} 
\frac{(300+s)^2}{2(600^2)} & \text{for } s \in [-300, 300] \\
1 - \frac{(900-s)^2}{2(600^2)} & \text{for } s \in [300, 900] 
\end{cases}
\]

One can determine the sellers’ pricing behavior with semi-private and private preferences using \( H(s) \), and with homogeneous preferences using \( F(c) \).

### 3.1 Price Comparisons

Figure 1 plots price-setting functions for each setting, for two sellers in panel (a) and four sellers in panel (b). For homogeneous preferences Figure 1 plots the analytically derived price-setting function, \( p^H(c) \). For semi-private preferences Figure 1 plots the analytically derived price-setting function, \( p^{SP}(c, v) \), for values of 300 and 900. These functions provide bounds on the price-setting functions with semi-private preferences, because \( p^{SP}(c, v) \) increases in \( v \). Figure 1 also plots the “average” price-setting function

\[
\bar{p}^{SP}(c) = \int_{300}^{900} p^{SP}(c, v) \, dG(v).
\]

\( \bar{p}^{SP}(c) \) is the seller’s expected price for each cost realization, where the expectation is taken with respect to the value draws. Finally, Figure 1 plots the numerically approximated price-setting function with private preferences, \( p^P(c) \). For both values of \( N \), \( p^H(c) < \bar{p}^{SP}(c) < p^P(c) \) for all \( c \).

The finding that \( p^H(c) < \bar{p}^{SP}(c) \) might naturally be considered a reflection of the conventional wisdom that introducing horizontal product differentiation softens price competition, but such a view obscures a more fundamental principle based on the strategic equivalence of the homogeneous and semi-private preference settings that was mentioned earlier: in both settings each seller is privately informed, relative to other sellers, about how much total surplus its trade with the buyer creates, and the sellers compete by offering some portion of that surplus to the buyer. In equilibrium a seller offers the expected value of the highest of its rivals’ surpluses, conditional on its own surplus being the highest. The semi-private preference setting is constructed so that the probability distribution of surplus is a mean-preserving spread of the distribution from the homogeneous preference setting. Consequently, with semi-private preferences a seller offers less of the available surplus to the buyer than it does in the analogous cases with homogeneous preferences. When translated into pricing terms, prices are higher on average in the semi-private than in the homogeneous preference setting. However, the fundamental point is that the same change in pricing would emerge if instead there had been an equivalent mean-preserving spread of the cost distribution in the homogeneous preference setting. What matters is the increased differentiation of the sellers in terms of the available surplus their trade with the buyer creates, not the introduction of product differentiation.
The finding that $p^{SP}(c) < p^{P}(c)$ results from the resolution of two conflicting forces stemming from the sellers’ increased uncertainty that they are the buyer’s surplus-maximizing trading partner, with private preferences. For a given cost, with semi-private preferences a seller that is a good fit with the buyer charges a relatively high price, while a seller that is a bad fit can remain competitive by charging a relatively low price. Rival sellers encounter similar incentives. With private preferences the sellers’ uncertainty about their fit with the buyer eliminates such fine-tuned pricing responses, and in the cases reported in Figure 1 price competition softens sufficiently that prices increase with private preferences. However, there is no guarantee this relationship holds for all distributions of costs and values.

Finally, the finding that $p^{H}(c) < p^{P}(c)$ results from combining the two effects just discussed. The change from homogeneous to private preferences can be decomposed into the change from homogeneous to semi-private preferences, which increases the variance of the probability distribution of surplus, and the change from semi-private to private preferences, which increases the sellers’ uncertainty that they are the buyer’s surplus-maximizing trading partner. However, there was reason to expect a different result. A seller’s beliefs that it is the buyer’s surplus-maximizing trading partner get “flatter” when moving from homogeneous to private preferences. That is, a low-cost seller is less sure it is the buyer’s surplus-maximizing trading partner, while a high-cost seller is less sure it is not. Consequently, with private preferences one might have expected a seller to set lower prices when its costs are low, and higher prices when its costs are high, compared to its behavior with homogeneous preferences. In fact, if there are circumstances in which the sellers’ uncertainty about their fit with the buyer causes $p^{H}(c) < p^{SP}(c)$, then it is conceivable price competition becomes sufficiently intense that $p^{P}(c) < p^{H}(c)$ for at least some values of $c$.

### 3.2 Payoff Comparisons

The price comparisons in Figure 1 illustrate how the sellers’ behavior differs across the three settings, but payoff comparisons are also relevant with product differentiation because one must consider the match quality between the buyer and the winning seller. After all, if prices increase when moving from homogeneous to semi-private preferences, then the buyer’s expected payoff may increase because greater total surplus is available with semi-private preferences.6 For each setting and value of $N$, Table 1 reports the buyer’s and a representative seller’s expected payoff, $\pi_B$ and $\pi_i$, along with their ratio.

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<th>$N = 2$</th>
<th>$N = 4$</th>
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<td></td>
<td>Homogeneous</td>
<td>Semi-Private</td>
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<tr>
<td>$\pi_B$</td>
<td>200</td>
<td>184.063</td>
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<tr>
<td>$\pi_i$</td>
<td>100</td>
<td>128.438</td>
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<td>$\pi_B/\pi_i$</td>
<td>2</td>
<td>1.433</td>
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For the buyer’s expected payoff $\pi_B$, the only consistent change is that $\pi_B$ falls when moving from semi-private to private preferences, which is an obvious consequence of the higher prices and poorer match quality that result with private preferences. While one might anticipate the buyer would receive an information rent when the information it shared with each seller $i$ became the buyer’s private information, that effect is

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6 With homogeneous preferences, the expected value of the highest of $N$ surplus draws is 400 with two sellers and 480 with four sellers. With semi-private and private preferences, the expected value of the highest of $N$ surplus draws is 440.938 with two sellers and 554.532 with four sellers, conditional on the highest surplus being positive.
swamped by strategic effects from eliminating the sellers’ fine-tuned pricing response to the buyer’s preferences. Table 1 also shows that $\pi_B$ can increase or decrease when moving from homogeneous to semi-private preferences, depending on the number of sellers. This result is driven entirely by properties of order-statistics, because in both settings the buyer’s expected payoff is the expected value of the second-highest of $N$ surplus draws from the associated cumulative distribution of total surplus. With the mean-preserving spread of total surplus associated with moving from homogeneous to semi-private preferences, the expected value of the second-highest surplus falls with two sellers, and rises with four sellers.

For a seller’s expected payoff $\pi_i$, the consistent change instead is that $\pi_i$ increases when moving from homogeneous to semi-private preferences. In both settings, each seller’s expected payoff is its share $\frac{1}{N}$ of the expected difference between the highest and second-highest of $N$ surplus draws. The increased variance of total surplus when moving from homogeneous to semi-private preferences increases the sellers’ expected payoffs, but Katzman, et al. [2011] demonstrate that this qualitative result need not be generally true with such mean-preserving spreads of total surplus. Table 1 also shows that $\pi_i$ can increase or decrease when moving from semi-private to private preferences. Despite the less intense competition that manifests as higher prices, the sellers can be harmed by the poorer match quality and lower efficiency that results when sellers are uncertain about the buyer’s preferences for their products.

Even the ratio $\frac{\pi_B}{\pi_i}$ shows no consistent pattern. It falls when moving from homogeneous to semi-private preferences, then increases a small amount or decreases a large amount, depending on $N$. All in all, despite the consistent price changes there is no clear pattern that the buyer does consistently better or worse when moving from homogeneous to semi-private to private preferences, either in absolute terms or relative to the sellers.

The quantitative and qualitative differences across the three settings that are reported in Figure 1 and Table 1 show that the nature of product differentiation affects procurement competition. One might have thought that the equilibrium price-setting functions with homogeneous or semi-private preferences would lead to outcomes qualitatively similar to those with private preferences. That they do not is unfortunate, because the derivation of analytic solutions for the sellers’ equilibrium strategies with private preferences remains an open problem for this empirically relevant setting.

### 3.3 Discussion

The results in Figure 1 and Table 1 illustrate several differences among the private, semi-private, and homogeneous preference settings, but what makes model specification especially important is when two models give different qualitative predictions regarding some measure of interest. In this subsection I describe such qualitative differences between the private preference setting and either the semi-private or homogeneous preference settings, to suggest one be cautious using one of the analytically convenient models when the private preference model is empirically appropriate.

As the point of departure, I suppose that the private preference setting best describes the actual strategic environment, then discuss what inferences would be drawn if one instead modeled the environment using the more tractable semi-private or homogeneous preference settings. All comparisons are based on the cost and value distributions used in the numerical results presented above.

Two general points are worth noting initially. First, outcomes with private preferences are occasionally inefficient, while those with semi-private or homogeneous preferences are always efficient. Hence, any attempt to assess efficiency using semi-private or homogeneous preferences will be incorrect. Second, all three settings give identical predictions regarding changes in the number of sellers: increasing the number
of sellers increases the buyer’s expected payoff, and decreases each seller’s expected payoff. Hence, assessing the effect of entry using the semi-private or homogeneous preference models will give qualitatively correct predictions about changes in the parties’ payoffs.

In what follows I consider the effects of changing two primitives in the models: the variance of the sellers’ cost draws, and the variance of the buyer’s value draws.

**Variance of the cost draws.** Consider the effect of reducing the variance of the sellers’ cost draws, by evaluating an extreme case in which there is no cost variation. Instead, each seller’s cost is commonly known to be 300, which is the expected value of a seller’s cost according to the initial parameter specification. Gal-Or, et al. [2007] use such a model to examine the buyer’s incentives to reveal to sellers its private information about its values for their products.\(^7\) Unfortunately, I cannot use their results because those authors implicitly assume the buyer is willing to make trades that give it a negative payoff, which differs from the approach I take in this paper. However, with suitable modification it is reasonably straightforward to derive the payoffs reported in Table 2, when the buyer’s values are uniformly distributed from 300 to 900.

With homogeneous preferences, the buyer’s payoff is 300 with either two or four sellers, the difference between its commonly known value of 600 and the sellers’ commonly known cost of 300. The sellers’ payoffs likewise are 0, because they essentially engage in one-shot Bertrand competition with homogeneous products. With semi-private preferences the parties’ payoffs are identical to their payoffs with homogeneous preferences under the initial parameter specification reported in Table 1. In both cases the total surplus of an arbitrary trade is uniformly distributed from 0 to 600.

Comparing the payoffs in Table 2 to those from Table 1 illustrates the change when the cost variation drops to zero.

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<td>Semi-Private</td>
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<tr>
<td>(\pi_B)</td>
<td>300</td>
<td>200</td>
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<td>(\pi_i)</td>
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If one uses either the semi-private or homogeneous preference model, then the predicted direction of change in the buyer’s expected payoff is incorrect with either two or four sellers. With two sellers the semi-private and homogeneous models predict the buyer’s expected payoff increases, while the private preference model predicts the buyer’s expected payoff decreases. With four sellers the semi-private and homogeneous models predict the buyer’s expected payoff decreases, while the private preference model predicts the buyer’s expected payoff increases. With either number of sellers it also is worth noting that the magnitude of the change in the buyer’s expected payoff is much greater in the homogeneous preference model.

All three models have the same prediction about the direction of change in the sellers’ expected payoffs, at least for the parameters used in deriving the payoffs reported in Tables 1 and 2. In each model a seller’s expected payoff falls as the variance of the cost draws drops to zero, which reflects a loss in the sellers’ information rents associated with their private information about their costs.

**Variance of the value draws.** Consider the effect of reducing the variance of the buyer’s value draws, by evaluating an extreme case in which there is no value variation. Instead, the buyer’s value for each

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\(^7\)The authors’ idea is that the buyer can reveal such information by specifying in detail its procurement requirements. If one takes a similar view in my strategic environment with private information about the sellers’ production costs, then an unexpected consequence is that the buyer has a greater incentive to provide such information as the number of sellers increases.
seller’s product is commonly known to be 600, which is the expected value of the buyer’s value according to the initial parameter specification. Perloff and Salop [1985] interpret such a change as a reduction in the intensity of the buyer’s preferences. As mentioned earlier, this setting corresponds to the homogeneous preference model, which is a special case of the private and semi-private models in which the variance of the value draws is zero.

Table 3 reports the payoffs for this setting. Comparing these payoffs to those from Table 1 illustrates the change when the value variation drops to zero.

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<td>$\pi_i$</td>
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If one uses the semi-private preference model, then the predicted direction of change in the buyer’s expected payoff is incorrect with four sellers. With two sellers the semi-private preference model predicts the buyer’s expected payoff increases, while with four sellers it decreases. With either two or four sellers the private preference model predicts the buyer’s expected payoff increases. Both models predict the same direction of change in the sellers’ expected payoffs: the sellers’ expected payoffs decrease when the variance of the value draws drops to zero.

If one uses the homogeneous preference model, then the predicted direction of change in the buyer’s and the sellers’ expected payoffs is incorrect with either two or four sellers. The homogeneous preference model has no value variation, and so it predicts no change in the buyer’s or the sellers’ expected payoffs. The private preference model predicts that the buyer’s expected payoff increases with either two or four sellers, and that the sellers’ expected payoffs decrease with either two or four sellers.

**Implications, and other comparative statics.** The preceding results demonstrate that in many respects the private preference setting is qualitatively different from either the semi-private or homogeneous preference settings. Such differences imply that one’s inferences can be affected by using the more tractable semi-private and homogeneous preference models when the private preference model is more appropriate.

For example, the different comparative statics for the buyer’s expected payoff affect assessments of how the buyer’s incentive to engage in procurement varies with changes in the degree of product differentiation, or in the variation among sellers’ costs. Conclusions regarding the buyer’s optimal number of sellers to engage in procurement are likewise affected. Similarly, the different comparative statics for the sellers’ expected payoffs affect inferences regarding sellers’ decision to incur any entry costs associated with a procurement contest, such as those reflecting prototype development or due diligence. Finally, all of the qualitative differences may affect the conclusions drawn from structural econometric techniques, such as those in Guerre, et al. [2000].

One could derive additional comparative statics by changing other aspects of the original model’s structure. For example, one could consider other changes in the variances of the sellers’ costs and the buyer’s values, as there is no guarantee the results described above hold in all cases. Likewise, one could examine changes in the mean of the cost or value draws, rather than the variance. Differences across the models in the latter respect could influence conclusions regarding issues such as the sellers’ incentives to invest in cost-reducing technology. Likewise, one could consider entirely different distributions of costs and values.
All of these possible extensions suggest circumstances in which having closed-form solutions for the private preference setting might be useful.

4 Insights Regarding Price Discrimination

Corts [1998] examines third-degree price discrimination using a general model of differentiated goods duopoly with atomistic buyers and complete information about all market parameters. He characterizes circumstances in which the use of price discrimination leads to all-out competition amongst the sellers that lowers their profits by causing prices for all products to decline. The presence of these unambiguous consumer welfare gains contrasts with the ambiguous effects that typically arise in monopoly models of third-degree price discrimination, in which discrimination leads to higher prices for consumers in the seller’s “strong market,” and to lower prices for consumers in the seller’s “weak market.”

A key ingredient of the unambiguous welfare effects is what Corts [1998] refers to as best-response asymmetry: Essentially the sellers disagree as to which market is their strong market, so unilateral departures from the non-discriminatory equilibrium entail opposing pricing responses from the two sellers. In the equilibrium with discrimination, the interaction of these opposing incentives can be sufficiently strong that prices rise for all products or fall for all products, with the sellers’ profits likewise rising or falling.

There is a linkage between my model of horizontal product differentiation and Corts’ model of price discrimination, because the semi-private and private preference settings can be interpreted as settings with and without third-degree price discrimination. With semi-private preferences a seller’s price to a buyer varies with the buyer’s value for the seller’s product, while with private preferences it does not. A version of best-response asymmetry arises for particular realizations of the buyer’s values, in the sense that the sellers have opposing unilateral pricing incentives when departing from non-discriminatory pricing.

My results in Section 3 demonstrate that Corts’ insights are relevant to strategic settings with large buyers and incomplete information, and they provide new insights regarding the effects of third-degree price discrimination. Table 1 reports that allowing price discrimination increases the buyer’s expected payoff with two or four sellers, which is reflective of Corts’ best-response asymmetry. However, the sellers are not necessarily harmed when discrimination is allowed, because the increased efficiency that discrimination creates can permit both the buyer and the sellers to gain. My results show that the effect of discrimination on the sellers’ payoffs is affected by incomplete information about the sellers’ costs and by the number of sellers. The latter effect is evident from the payoffs reported in Table 1 and from the price-setting functions plotted in Figure 1: with four sellers the price-setting function with private preferences is nearly as high as the one with semi-private preferences for the highest possible buyer’s value. This effect of the number of sellers is not apparent from Corts [1998], which focused on duopoly. Finally, the ambiguous effects from price discrimination also likely depend on the distributions of the sellers’ costs and the buyer’s values.

5 Conclusion

This paper illustrates quantitative and qualitative effects on procurement outcomes that arise from the nature of horizontal product differentiation, in terms of what sellers know about the buyer’s preferences. Sellers’ equilibrium prices increase when moving from homogeneous to semi-private to private preferences, for the parameters I consider, but the buyer’s and the sellers’ expected payoffs exhibit no such consistent pattern of change across the three settings. Moreover, payoffs in the models exhibit qualitatively different
comparative statics, which affects a host of predictions and prescriptions regarding equilibrium behavior.

The differences among the homogeneous, semi-private, and private preference settings caution against using either of the analytically convenient models when the private preference model is empirically appropriate, and more generally indicate the importance of appropriately specifying the nature of the information available to market participants. Because the setting with private preferences has not yielded analytic solutions, efforts to find such solutions would be worthwhile given the setting’s empirical relevance and the qualitative difference in outcomes from settings for which analytic solutions are available. Having closed-form solutions would enable assessment of these results’ generality, and of standard topics such as horizontal mergers, entry, and investment in cost reductions.

6 Appendix

This appendix describes the numerical techniques used to approximate the sellers’ equilibrium price-setting function with private preferences. The number of sellers is 2 or 4, the sellers’ costs are Uniformly distributed from 0 to 600, and the buyer’s values are Uniformly distributed from 300 to 900.

Solving for a symmetric equilibrium from the perspective of seller 1, the iterative approach begins by finding seller 1’s best-response to an initial price-setting function used by seller 1’s rivals, \( p_0 (c) \). Call this best-response \( p_1 (c) \). The iterative procedure continues with seller 1’s rivals using \( p_1 (c) \), then deriving seller 1’s best-response to that, \( p_2 (c) \). The procedure stops at step \( t \) when the maximum distance between \( p_{t-1} (c) \) and \( p_t (c) \) is less than a user-specified tolerance. The final best-response function, \( p_t (c) \), is the approximated equilibrium price-setting function.

Given how seller 1’s rivals set prices, seller 1’s best-response for each cost \( c \) is the price \( p \) that maximizes \( (p - c) \Pr [p \text{ wins}] \). The probability function depends on the rivals’ price-setting function, and that probability function’s nature creates the difficulty in deriving analytic equilibrium solutions. Roughly speaking, with homogeneous or semi-private preferences the probability a particular price wins is a length, while with private preferences the probability is an irregularly shaped volume.

I approximate the probability function in seller 1’s objective function in the following manner. Given the rivals’ price-setting function, I take two million random draws of values and costs for each rival, and of values for seller 1. I discretize the price space, and for each price \( p \in \{0, 1, 2, \ldots, 900\} \) the probability that price wins is the fraction of the two million cases in which with price \( p \) seller 1 offers the buyer a positive payoff that exceeds the payoff from the rivals.

I approximate seller 1’s best-response to its rivals’ price-setting function in the following manner. I discretize the cost space, and for each cost \( c \in \{0, 5, 10, \ldots, 600\} \) I find the price \( p \in \{0, 1, 2, \ldots, 900\} \) that gives seller 1 the highest expected payoff.

Having found seller 1’s best-response at a discrete set of costs, I fit a cubic to those points. The fitted cubic is set as the rivals’ price-setting function, and I repeat the procedure described above. I use the same set of value and cost draws for each iteration. The procedure concludes when the maximum difference between the rivals’ price-setting function and seller 1’s best-response is less than 1, evaluated at each cost \( c \in \{0, 5, 10, \ldots, 600\} \). The procedure converged in fewer than 10 iterations, and with different initial price-setting functions the process converged to approximately the same price-setting function.

With the approximated price-setting function in hand, I used the same random draws to calculate the buyer’s and sellers’ expected payoffs.
References


Figure 1. Equilibrium price-setting functions with homogeneous preferences (H), semi-private preferences (SP), and private preferences (P). Each curve represents a seller's price as a function of its cost realization. For the semi-private setting, functions are plotted for two specified values, as is the "average" price-setting function. Panel (a) represents the two-seller environment, and panel (b) represents the four-seller environment.